Param Pujya Dr. Babasaheb Ambedkar Smarak Samiti's
Dr. Ambedkar Institute of Management Studies & Research
Deeksha Bhoomi, Nagpur - 440010 (Maharashtra State) INDIA
NAAC Accredited with 'A' Grade
Tel: +91 712 6521204, 6521203 ,6501379
Email: info@daimsr.in

Param Pujya Dr. Babasaheb Ambedkar Smarak Samiti's
Dr. Ambedkar Institute of Management Studies & Research
Deeksha Bhoomi, Nagpur - 440010 (Maharashtra State) INDIA
NAAC Accredited with 'A' Grade
Tel: +91 712 6521204, 6521203 ,6501379
Email: info@daimsr.in

Unit III: Assignment

Programme Educational Objectives

Our program will create graduates who:

1. *Will be recognized as a creative and an enterprising team leader.*
2. *Will be a flexible, adaptable and an ethical individual.*
3. *Will have a holistic approach to problem solving in the dynamic business environment.*

Operations Research Course Outcomes

- CO1-Given a verbal descriptive problem (management, industry or miscellaneous) with numerical data, the student manager will be able to define the variables, establish the inter-relationships between them, formulate the objective function and constraints and solve the problem graphically for optimization.
- CO2-Given/ specified the competition scenario between two players and their payoffs in advance, the student manager will be able to identify the saddle point and/ or determine the optimum strategies of both the players that would result in optimum payoff (gain or loss)to both the players.
- CO3-Given a set of limited resources, a set of limited activities and related cost/ profit matrix, the student manager will be able to assign one resource to one activity so as to maximize or minimize the given measure of effectiveness.

CO4-Given a business situation containing the transportation costs from n sources to m destinations, the student manager will be able to associate one source to one destination to minimize the cost of transportation.

CO5-In a decision making environment that is represented by numerical data, the student manager will be able to apply relevant operations research technique for managerial decision making and problem solving.

Syllabus

- Characteristics of assignment problem
- Assumptions made in assignment problem
- Formulation of assignment problem
- Hungarian method
- Maximization problems
- Restricted Assignment problem

Unit Objectives

- To understand the concept of assignment
- To understand the Formulation of Assignment pro
- To practice the problems on assignment (maximization problem) through Hungarian Method

Content

- Characteristics of assignment problem
- Assumptions made in assignment problem
- Formulation of assignment problem
- Hungarian method
- Maximization problems
- Restricted Assignment problem

- The assignment problem is a special type of linear programming problem.
- The problem of assignment arises because of limited resources.
- In real life, we are faced with the problem of allocating different personnel/ workers to different jobs. Not everyone has the same ability to perform a given job. Different persons have different abilities to execute the same task and these different capabilities are expressed in terms of cost/profit/time involved in executing a given job. Therefore, we have to decide: How to assign different workers to different jobs" so that, cost of performing such job is minimized.

Applications

- Assigning workers to offices or jobs
- Assignments of trucks to delivery routes
- Assignments of drivers to trucks
- Assignments of classes to rooms
- Assignments of Salesmen to different sales areas
- Assignments of clerks to different checkout counters
- Assignments of different machines to different workers

Characteristics/ Assumptions Of An Assignment Problem:

1. Square matrix: The number of assignees and number of task are the same (this number is denoted by n).
2. 1A-1T: Each assignee is to be assigned to perform exactly one task/ Each task is to be performed by exactly one assignee.
3. Payoff : There is a cost or profit associated with assignees performing different task.
4. The objective is to determine how all n assignment should be made to optimize the given pay offs which are expressed in terms of cost, time spent, distance, revenue earned, production obtained etc.

Example A1

A computer centre has three expert programmers. The Centre wants three application programmes to be developed. The head of the computer centre , after studying carefully the programmes to be developed, estimates the computer time in minutes req. by the experts for the application programmes as follows. Assign the programmer to the programmes in such a way that the total computer time is minimum.

	Programmer			
Programmes		**A**	**B**	**C**
	1	**120**	**100**	**80**
	2	**80**	**90**	**110**
	3	**110**	**140**	**120**

Example A2

A department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

	Employees					
Jobs		I	II	III	IV	V
	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hour?

Example A2

Solve the assignment problem

	Employees					
Jobs		I	II	III	IV	V
	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	13

Example A3

A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimates of the times that each man would take to perform each task is given below. How should the tasks be allocated to subordinates so as to minimize total man-hours?

	Tasks				
Subordinates		I	II	III	IV
	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

Example A4

A pharmaceutical company is producing a single product and is selling it through five agencies situated in different cities. All of a sudden, there is a demand for the product in another five cities not having any agency of the company. The company is faced with the problem of how much to assign the existing agencies to dispatch the product to needy cities in such a way that the travelling distance is minimized. The distance between surplus and deficit cities (in km) is given in the following table.

	Deficit cities					
Surplus cities		a	b	c	d	e
	A	160	130	115	190	200
	B	135	120	130	160	175
	C	140	110	125	170	185
	D	50	50	80	80	110
	E	55	35	80	80	105

Determine the optimum assignment schedule.

Example A5

A solicitors firm employs typists on hourly piece rate basis for their daily work. There are five typists and their charges and speeds are different. According to an understanding only one job is given to one typist and the typist is paid for a full hour even if he works for a fraction of an hour. Find the least cost allocation for the following data.

Typist	Rate per hour (Rs)	No. of pages types /Hour
A	5	12
B	6	14
C	3	8
D	4	10
E	4	11

Job	No. of pages
P	199
Q	175
R	145
S	298
T	178

Multiple optimal solutions Maximization case Example A6

The maximum sales matrix is given below. Assign the salesman to the territory to maximize the sales.

	Territory				
Sales man		I	II	III	IV
	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Example A7

A marketing manager has five sales districts. Considering the capabilities of the salesmen and the nature of districts, the marketing manager estimates the sales per month (in hundred rupees) for each salesman in each district would be as follows. Find the assignment of salesmen to districts that will maximize the sales.

	Districts					
Salesman		I	II	III	IV	V
	A	32	38	40	28	40
	B	40	24	28	21	36
	C	41	27	33	30	37
	D	22	38	41	36	36
	E	29	33	40	35	39

A8

The Spicy Spoon restaurant has four payment counters. There are four persons available for service. The cost of assigning each person to each counter is given in the following table.

Assign one person to one counter to minimize the total cost.

	Job			
Person	**1**	**2**	**3**	**4**
A	**1**	**8**	**15**	**22**
B	**13**	**18**	**23**	**28**
C	**13**	**18**	**23**	**28**
D	**19**	**23**	**27**	**31**

Example A9

- A project work consists of four major jobs for which an equal no. of contractors have submitted tenders. The tender amount quoted 9in lakhs of rs) is given in the matrix. Find the assignment which will minimize the total cost of the project.

	Jobs				
Contractors		a	b	c	d
	1	10	24	30	15
	2	16	22	28	12
	3	12	20	32	10
	4	9	26	34	16

Unbalanced assignment problem Restrictions on assignment

- The Hungarian method of assignment requires number of rows and columns in the assignment matrix to be equal. However, when the given cost matrix is not a square matrix, the assignment problem is called as an **unbalanced problem.** In such cases dummy rows or dummy columns are added in the matrix to make it a square matrix.

- Sometimes it may happen that a particular resource cannot be assigned to perform a particular activity. In such cases, the cost of particular activity by a particular resource is considered to be very large (written as M or ∞) so as to prohibit the entry of this pair of resource – activity into the final solution.

Example A10

In the modification of a plant layout of a factory four new machines M1, M2, M3 and M4 are to be installed in a machine shop. There are five vacant places A, B, C, D, and E available. Because of limited space, machine M2 cannot be placed at C and m3 cannot be placed at A. The cost of locating a machine at a place (in hundred rupees is a follows. Find the optimal assignment schedule.

	Location					
Machine		A	B	C	D	E
	M1	9	11	15	10	11
	M2	12	9	--	10	9
	M3	--	11	14	11	7
	M4	14	8	12	7	8

Example A11

An airline company has drawn up a new flight schedule involving five flights. To assist in allocating five pilots to the flights, it has asked them to state their preference scores by giving each flight a number out of 10. the higher the number, the greater is the preference. Certain of these flights are unsuitable to some pilot owing to domestic reasons. These have been marked with a x. what should be the allocation of the pilots to flights in order to meet as many preferences as possible?

	Flight Number					
Pilots		1	2	3	4	5
	A	8	2	X	5	4
	B	10	9	2	8	4
	C	5	4	9	6	X
	D	3	6	2	8	7
	E	5	6	10	4	3

Example A12

A city corporation has decided to carry out road repairs on main four arteries of the city. The govt has agreed to make a special grant of Rs 50 lakhs towards the cost with a condition that the repairs be done at the lowest cost and quickest time. If the conditions warrant, a supplementary token grant will also be considered favourably. The corporation has floated tenders and five contractors have sent in their bids.

a. Find the best way of assigning work.

b. B. if it is necessary to seek grant, what should be the amount?

c. Which of the five contractors will be unsuccessful in his bid?

	r1	r2	r3	r4
C1	9	14	19	15
C2	7	17	20	19
C3	9	18	21	18
c4	10	12	18	19
c5	10	15	21	16

Special cases
Travelling Salesman Problem A13

A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost in Rs ‘000 of each city from a particular city is given below.

What is the sequence of the visit of the salesman so that the cost is minimum?

	To City					
From City		A	B	C	D	E
	A	-	2	5	7	1
	B	6	-	3	8	2
	C	8	7	-	4	7
	D	12	4	6	-	5
	E	1	3	2	8	-

A14

A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling distance of each city from a particular city is given below.

The Salesman starts from A and comes back to A. What route should he follow so that the distance is minimum?

	To City					
From City		A	B	C	D	E
	A	-	1	6	8	4
	B	7	-	8	5	6
	C	6	8	-	9	7
	D	8	5	9	-	8
	E	4	6	7	8	-

A16

The expected times required to be taken by a salesman in travelling from one city to another are as follows.

How should the salesman plan his trip so that he covers each of these cities no more than once, and completes his trip in minimum possible time required for travelling?

		To City				
		C1	C2	C3	C4	C5
From City	C1	-	10	13	11	-
	C2	10	-	12	10	12
	C3	14	13	-	13	11
	C4	11	10	14	-	10
	C5	12	11	12	10	-

A17

Solve the travelling salesman problem given by the following data.

C12=20, C13=4, C14=10, C23=5,

C24=6, C25=10, C35=6, C45=20

Where Cij=Cji

And there is no route between cities I and j if the values for Cij is not given.

Assessment Questions

- What is an assignment problem? Give two applications.
- Explain the Hungarian method of Assignment.
- Explain the conceptual justification that an assignment problem can be viewed as a LPP.
- Explain the difference between Tx problem and Assignment Problem.
- Give the algorithm to solve an assignment problem.
- Characteristics of assignment problem
- Assumptions made in assignment problem
- What is multiple option solution in AP?
- Can there be multiple optimal solutions to an assignment problem? How would you identify the existence of multiple solutions , if any?
- How do you deal with the problems where the objective function is to be maximizes?
- What is unbalanced AP? How is solution obtained in such case?

Reference Books

- Operations Research Theory and Applications by J.K. Sharma, Macmillan India Ltd.