Linear Programming Problems

Programme Educational Objectives

Our program will create graduates who:

1. *Will be recognized as a creative and an enterprising team leader.*
2. *Will be a flexible, adaptable and an ethical individual.*
3. *Will have a holistic approach to problem solving in the dynamic business environment.*

Operations Research Course Outcomes

- CO1-Given a verbal descriptive problem (management, industry or miscellaneous) with numerical data, the student manager will be able to define the variables, establish the inter-relationships between them, formulate the objective function and constraints and solve the problem graphically for optimization.
- CO2-Given/ specified the competition scenario between two players and their payoffs in advance, the student manager will be able to identify the saddle point and/ or determine the optimum strategies of both the players that would result in optimum payoff (gain or loss)to both the players.
- CO3-Given a set of limited resources, a set of limited activities and related cost/ profit matrix, the student manager will be able to assign one resource to one activity so as to maximize or minimize the given measure of effectiveness.

CO4-Given a business situation containing the transportation costs from n sources to m destinations, the student manager will be able to associate one source to one destination to minimize the cost of transportation.

CO5-In a decision making environment that is represented by numerical data, the student manager will be able to apply relevant operations research technique for managerial decision making and problem solving.

Unit Syllabus

- **LPP:** Application of Linear programming, General statement and assumptions underlying Linear Programming, Formulation of Linear Programming Problems,
- Graphical method for solution of LPP. Unbounded and degenerate solution of LPP
- **Game Theory:**
- Game models, Two persons zero sum games and their solution, solution of 2Xn and mX2 games by
- graphical approach, Solution of mXn games.

Unit Objectives

- To understand the concept of Linear Programming problems, general structure of LP model and graphical method for LPP solution
- To make the students understand the concept of theory of games, rules of game theory.

Introduction to Linear Programming

- A Linear Programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints.
- The linear model consists of the following components:
  - A set of decision variables.
  - An objective function.
  - A set of constraints.

Applications of LPP

Industrial Applications:

- Product mix problems
- Blending problems
- Production Scheduling problems
- Assembly line balancing
- Make-or-buy problems

Management Applications

1. Media Selection Problems
2. Portfolio selection Problems
3. Profit planning Problems
4. Transportation Problems
5. Assignment Problems
6. Manpower scheduling Problems
7. Plant location selection Problems

Example 1: Production Allocation

A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (minutes)			Machine Capacity (Minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (LP) model that will maximize the daily profit.

Example 2: Diet problem

A Person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the table.

Food type	Yield per unit			Cost per unit (Rs.)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate linear programming model for the problem.

Example 4: Production allocation

- Company manufactures 2 types of gifts made of plywood. Gift A requires 5 minutes for cutting and 10 minutes for assembling. Gift B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. Profit of 50 paisa on Gift A and 60 paisa on Gift B can be made. How many gifts of each type should company manufacture in order to maximize profit. Formulate the above as a LPP

Example 5: Product Mix problem

- A dealer deals in only two items; Sewing machines and Table fans. He has Rs 25000 to invest and place to store at most 100 pieces. A sewing machine costs him Rs 400 and a table fan Rs. 200. He can sell a sewing machine at a profit of Rs 50 and table fan at a profit of Rs 30.
- Formulate as a linear programming model for determining how he should invest his money in order to achieve maximum profit.

Example 6: Advertising media selection problem

An advertising company wished to plan its advertising strategy in three different media-Television, radio and magazines. The purpose of advertising is to reach as large a number of potential customers as possible. Following data have been obtained from market survey:

	Television	Radio	Magazine 1	Magazine 2
Cost of an advertising unit	Rs. 30,000	Rs. 20,000	Rs. 15,000	Rs. 10,000
No. of potential customers reached per unit	2,00,000	6,00,000	1,50,000	1,00,000
No. of female customers reached per unit	1,50,000	4,00,000	70,000	50,000

Cont...

The company wants to spend not more than Rs. 4,50,000 on advertising. Following are the further requirements that must be met:

i. At least 1 million exposures take place among female customers.

ii. Advertising on magazines be limited to Rs. 1,50,000.

iii. At least 3 advertising units be bought on magazine 1 and 2 units on magazine 2.

iv. The number of advertising units on television and radio should each be between 5 and 10.

Formulate the LPP model.

Example 7: Flight Scheduling problem

An aircraft company, which operates out of a central terminal has 8 aircrafts of type I, 15 of type II and 12 aircrafts of type III available for today's flight. The tonnage capacities (in thousands of tons) are 4.5 for type I, 7 for type II and 4 for type III.

The company dispatches its planes to cities A and B. Tonnage requirements are 20 (in thousands of tons) at city A and 30 at city B. Excess tonnage capacity supplied to a city has no value. A plane can fly once only during the day.

The cost of sending a plane from the terminal to each city is given by the following table.

	Type I	Type II	Type III
City A	23	5	1.4
City B	58	10	3.8

Formulate the LP model.

Example 8

- A company produces two types of leather belts, say types A and B. Belt A is of superior quality and belt B is of inferior quality. Profit on the two types of belts are 40 and 30 paisa per belt respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all belts were of type B, the company could produce 1000 belts/day. The supply of leather, however is sufficient only for 800 belts per day. Belt A requires a fancy buckle and only 400 fancy buckles are available for this per day. For B 700 buckles are available per day. How should the company manufacture the two types of belts in order to have a maximum overall profit.

Graphical method of LPP

Example 9 (2.9.1)

- A firm manufactures two products A and B on which the profits earned per unit are Rs. 3 and Rs. 4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2. Machine M1 is available for not more than 7 hrs. 30 mins. While machine M2 is available for 10 hrs. during any working day. Find the number of units of products A and B to be manufactured to get maximum profit.

Example 10 (2.9.4)

- **Maximize $Z=3x1 + 4x2$ Subject to**

  $5x1+ 4x2<=200,$

  $3x1+ 5x2<=150,$

  $5x1 + 4x2>=100,$

  $8x1 + 4x2>=80,$

  $x1, x2>=0.$

Example 11

- The ABC Electric Appliance Company produces two products: Refrigerators and Ranges. Production takes place in two separate departments. Refrigerators are produced in department I and ranges are produced in dept II. Company's two products are produced and sold on weekly basis. Weekly production cannot exceed 25 refrigerators in dept I and 35 ranges in dept II. The company regularly employs a total of 60 workers in the two depts. A refrigerator requires 2 man-week of labour and ranges require 1 man week of labour. A refrigerator requires a profit of Rs. 60 and a range contributes a profit of Rs. 40. how many units of refrigerators and ranges should a company produce to realize a maximum profit?

Example 11 (2.9.3)

- **Maximize $Z=2x1 + x2$ Subject to**

  $x1+ 2x2<=10,$

  $x1+ x2<=6,$

  $x1 - x2<=2,$

  $x1 - 2x2<=1,$

  $x1, x2>=0.$

Example 12 (2.9.2)

Maximize $Z=2x1 + 3x2$

Subject to

$x1+ x2<=30,$

$x2>=3,$

$x2<=12,$

$x1 – x2>=0,$

$0<=x1<=20,$

$x1>=0,$

$0<=x1<=20.$

Example 13 - Minimization

Mohan Meakins Breveries Ltd has two bottling plants, one located at Solan and the other at Mohan Nagar. Each plant produces three drinks, whisky, beer and fruit juices named A, B and C resp. The number of bottles produced per day are as follows:

	Solan	Mohan Nagar
Whiskey, A	1,500	1,500
Beer, B	3,000	1,000
Fruit Juices, C	2,000	5,000

A market survey indicates that during the month of April, there will be a demand of 20,000 bottles of whisky, 40,000 bottles of beer and 44,000 bottles of fruit juices. The operating costs per day for plants at Solan and Mohan Nagar are 600 and 400 monetary units. For how many days each plant be run in April so as to minimize the production cost, while still meeting the market demand?

Example 14

Find the minimum value of

$$Z=5x1 - 2x2$$

Subject to

$$2x1 + 3x2 >= 1,$$

$$x1, x2 >= 0.$$

Example 15

Find the minimum value of

$$Z=3x1 + 4x2$$

Subject to

$$-x1 + 3x2<=10,$$

$$x1 + x2<=6,$$

$$x1 – x2<=2,$$

$$x1, x2>=0.$$

Example 16

A feed mixing operation can be described in terms of two activities. The required mixture must contain four kinds of ingredients *w*, *x*, *y* and *z*. Two basic feeds A and B, which contain the required ingredients are available in the market,1 kg of A contains 0.1 kg of w, 0.1 kg of y and 0.2 kg of z. Likewise, 1 kg of feed B contains 0.1 kg of x, 0.2 kg of y and 0.1 kg of z. the daily per head requirement is of at least, 0.4 kg of w, 0.6 kg of x, 2 kg of y and 1.6 kg of z. Feed A can be bought for Rs 0.07 per kg and B for 0.05 per kg. Determine the quantity of feeds A and B in the mixture in order that the total cost is minimum.

Practice problem

The N. Discovery Corporation produces two products: I and II. The raw material requirements, space needed for storage, production rates, and selling prices for these products are given in Table 1.

TABLE .1 Production Data for N. Dustrious Company

	Product	
	I	II
Storage space (ft^2/unit)	4	5
Raw material (lb/unit)	5	3
Production rate (units/hr)	60	30
Selling price ($/unit)	13	11

The total amount of raw material available per day for both products is 1575 lb. The total storage space for all products is 1500 ft^2, and a maximum of 7 hours per day can be used for production. All products manufactured are shipped out of the storage area at the end of the day. Therefore, the two products must share the total raw material, storage space, and production time. The company wants to determine how many units of each product to produce per day to maximize its total income.

Practice problem

A company produces two parts P1and P2 used in television sets. A unit of P1 costs the company Rs. 5 in wages and Rs. 6 in material, while a unit of P2 costs the company Rs. 20 in wages and Rs. 10 in material. The company sells both parts on one-period credit term, but the company's labour and material expenses must be paid in cash. The selling price of P1 is Rs. 25/unit and P2 is Rs 60/unit. The company's production capacity is limited by two considerations. First, at the beginning of the period, the company ha an initial balance of Rs. 35,000. Second, the company has available in each period 1,600 hours of machine time and 1,400 hours of assembly time. The production of each P1 requires 2 hours of machine time and 1.5 hours of assembly time, while production of P2 requires 2 hours of machine time and 3 hours of assembly time. How would company maximize the total profit?

Problem from Assignment

- A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B. At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours. The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximize the combined sum of the units of X and the units of Y in stock at the end of the week.
- Formulate the problem of deciding how much of each product to make in the current week as a linear program.
- Solve this linear program graphically.

Characteristics of LPP

(a) Objective function

(b) Constraints

(c) Non-negativity

(d) Linearity

(e) Finiteness

Assumptions in LPP

- **Proportionality-** change in the constraint inequalities will have the proportional change in the objective function. 1 product - profit 10 Rs, 2-20
- **Additivity- Continuity-** Variables can take continuous values. (M1- t1. M2- t2)
- **Certainty-** profit per unit of product, availability of material and labor per unit, requirement of material and labor per unit are known
- **Finite choices-** the decision maker has certain choices

Limitations

1. Cannot solve qualitative variables
2. In some cases, confusing and misleading. For example 1.6 machines.
3. Cannot solve business problems of non-linear nature.
4. The factor of uncertainty is not considered.
5. Completely mathematical, complicated, trained personnel
6. Use of computers must for complex, large programs
7. Formulating the objective function is sometimes difficult.

GAME THEORY

Rules

- There are two teams – A and B – who will play 10 rounds of competition.
- You will choose to play either Red or Blue.
- You will be scored as per the Score Table.
- The first 8 rounds are a maximum of 3 minutes each.
- You can have a conference, via representatives, with your opposing group on fourth round (however, this can only take place at the request of both groups).
- You can have another conference (for a maximum of 3 minutes) on eighth round, if both groups choose this.
- The ninth and tenth rounds score double and you will have 5 minutes in each round to make your decision.
- If both groups play Blue, each scores '-6'
- If one group plays Blue, the other Red then Red = -12 and Blue = +12
- If both play Red, each scores '+6'

Group A	Group B	Score A	Score B
Red	Red	+3	+3
Red	Blue	-6	+6
Blue	Red	+6	-6
Blue	Blue	-3	-3

PLOT

- Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to: betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent.

Each prisoner is told the following:

- If A and B each betray the other, each of them serves 2 years in prison
- If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa)
- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)

Because betraying a partner offers a greater reward than cooperating with them, all purely rational self-interested prisoners would betray the other, and so the only possible outcome for two purely rational prisoners is for them to betray each other. The interesting part of this result is that pursuing individual reward logically leads both of the prisoners to betray, when they would get a better reward if they both kept silent.

Game Theory

- Theory of games, competitive strategies.
- Mathematical theory used in competitive situations.
- When two or more individuals or organizations with conflicting objectives try to make decisions.
- A decision made by one decision maker affects the decision made by one or more of the remaining decision makers and the final outcome depends upon the decision of all the parties.

Applications

- Two players struggling to win chess
- Tic tac tow
- Candidates fighting elections
- Two enemies planning war tactics

Managerial/ business applications

- Firms struggling to maintain market share
- Launching advertisement campaigns.
- Negotiations, auctions

Principles of Game theory

Minimax principle

Best from the worst

Note: This theory does not describe how a game should be played; it describes only the procedures and principles by which plays should be selected.

Characteristics of Games

- There are finite number of competitors known as **'players'**
- Each player has a limited/ finite number of possible **courses of action** known as **'strategies'**
- All the strategies and their impacts are specified to the players but player does not know which strategy is to be selected.
- A game is played/ play is said to have occured when every player selects one of his strategies. The strategies are supposed to be prepared simultaneously with an outcome such that no player recognizes his opponent's strategy until he chooses his own strategy.
- Each combination of courses of action determines an outcome which results in gains to the participants. The gain can be positive or negative. Negative gain is called loss.

Characteristics of Games

- The gain of a participant depends not only on his own action but also those of others.
- The gains (payoffs) for each and every play are fixed and specified in advance and are known to each player.
- The players make individual decisions without direct communication

Definitions

- The player playing the game always attempts to select the best course of action which results in optimal pay off known as **'optimal strategy'**.
- The expected pay off when all the players of the game go after their optimal strategies is called as 'value of the game'. The main aim of a problem of a game is to determine the **value of the game.**
- The game is said to be 'fair' if the value of the game is zero or else it s known as '**unfair**'.

Important terms

1. Game: it is an activity between two or more players involving actions by each one of them according to set rules.
2. Player: each participant or competitor playing a game is called a player.
3. Play: a play of the game is said to occur when each player chooses one of his courses of action.
4. Strategy: it is a predetermined rule by which a player decides his course of action from his list .
5. Pure strategy:
6. Mixed strategy
7. Optimal strategy: that strategy that puts the player in the most preferred position irrespective of the strategy of his opponents. Highest payoff
8. Zero sum game: it is the game in which the sum of payments to all the players after the play of the game is zero.
9. Payoff: outcome of the game.
10. The figures present as the outcomes of strategies in a matrix form are known as '**pay-off matrix**'.
11. Saddle point/ stable game

$$
\begin{array}{c|c}
& \text{Player } B \\
& \begin{array}{cccccc} 1 & 2 & 3 & \cdots & j \cdots & n \end{array} \\
\text{Player } A \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ i \\ \vdots \\ m \end{array} &
\left[\begin{array}{cccccc}
a_{11} & a_{12} & a_{13} & \cdots & a_{1j} \cdots & a_{1n} \\
a_{21} & a_{22} & a_{23} & \cdots & a_{2j} \cdots & a_{2n} \\
a_{31} & a_{32} & a_{33} & \cdots & a_{3j} \cdots & a_{3n} \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
a_{i1} & a_{i2} & a_{i3} & \cdots & a_{ij} \cdots & a_{in} \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} \cdots & a_{mn}
\end{array}\right]
\end{array}
$$

A's payoff matrix

TABLE 9.79

$$
\begin{array}{c|c}
& \text{Player } B \\
& \begin{array}{ccccc} 1 & 2 & 3 & \ldots j & \ldots n \end{array} \\
\text{Player } A \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ i \\ \vdots \\ m \end{array} &
\left[\begin{array}{ccccc}
-a_{11} & -a_{12} & -a_{13} & \cdots -a_{1j} & \cdots -a_{1n} \\
-a_{21} & -a_{22} & -a_{23} & \cdots -a_{2j} & \cdots -a_{2n} \\
-a_{31} & -a_{32} & -a_{33} & \cdots -a_{3j} & \cdots -a_{3n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
-a_{i1} & -a_{i2} & -a_{i3} & \cdots -a_{ij} & \cdots -a_{in} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
-a_{m1} & -a_{m2} & -a_{m3} & \cdots -a_{mj} & \cdots -a_{mn}
\end{array}\right]
\end{array}
$$

B's payoff matrix

Rules for playing the game

- Each player has finite strategies.
- A is the gainer, B looser.
- A attempts to maximize his minimum gains- Maxmin principle
- B attempts to minimize his maximum losses-Minmax principle
- The sum of payoff matrices of A and B is a null matrix.
- Here the objective is to determine the optimum strategies of both the players that result in optimum payoff to each, irrespective of the strategy used by the other.
- Find saddle point

Example 1:

For the game with payoff matrix:

		PLAYER B	
		B1	B2
PLAYER A	A1	4	6
	A2	3	5

Example 2:
Rule 1: game with saddle point
For the game with payoff matrix:

		PLAYER B		
		B1	B2	B3
PLAYER A	A1	-1	2	-2
	A2	6	4	-6

Determine the best strategies for players A and B.
Also determine the value of game.
Is the game fair?
Is the game strictly determinable?

Example 3:

A company management and the labour union are negotiating a new three year settlement. Each of these has 4 strategies.

I: Hard and aggressive bargaining

II: Reasoning and logical approach

III: Legalistic strategy

IV: Conciliatory approach

The costs to the company are given for every pair of strategy choices.

Union Strategies	Company Strategies			
	I	II	III	IV
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	-5	4	11	0

What strategy will the two sides adopt? Also determine the value of the game.

Consider the game with the following payoff:

	Player B		
Player A		B1	B2
	A1	2	6
	A2	-2	λ

1. Show that G is strictly determinable, whatever λ may be.
2. Determine the value of G.

For what value of λ, the game with following payoff matrix is strictly determinable?

	B1	B2	B3
A1	λ	6	2
A2	-1	λ	-7
A3	-2	4	λ

Find the ranges of values of p and q which will render the entry (2, 2) a saddle point for the game.

	B1	B2	B3
A1	2	4	5
A2	10	7	q
A3	4	p	6

The payoff of a game is given below. Find the solution of the game to A and B.

		B				
		I	II	III	IV	V
A	I	-4	-2	-2	3	1
	II	1	0	-1	0	0
	III	-6	-5	-2	-4	4
	IV	3	1	-6	0	-8

Ans: Value of the game is -1 to A and +1 to B.

The payoff of a game is given below. Find the solution of the game to A and B.

	B					
		I	II	III	IV	V
A	I	3	-1	4	6	7
	II	-1	8	2	4	12
	III	16	8	6	14	12
	IV	1	11	-4	2	1

- Two competitive manufacturers are producing a new toy under license from the patent holder. In order to meet the demand they have the option of running the plant for 8, 16 or 24 hours a day. As the length of production increases so does the cost. One of the manufacturers say A, has set up the matrix given below, in which he estimates the percentage of the market that he could capture and maintain the different production schedules:

	Manufacturer B		
Manufacturer A	C1:8 Hrs	C2: 16 Hrs	C3: 24 Hrs
S1: 8 Hrs	60%	56%	34%
S2: 16 Hrs	63%	60%	55%
S3: 24 Hrs	83%	72%	60%

1. At what level should each produce?
2. What percentage of market will B have?

Assume that two firms are competing for market share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. Assume that the following payoff matrix describes the increase in market share of firm A and the decrease in market share of the firm B. determine the optimal strategies for each firm.

	Firm B		
Firm A	No Promotion	Moderate promotion	Much promotion
No Promotion	5	0	-10
Moderate promotion	10	6	2
Much promotion	20	15	10

i. Which firm would be the winner in terms of market share?

ii. Would the solution strategies necessarily maximize profits for either of the firms?

iii. What might the two firms do to maximize their profits?

Without saddle point
Mixed strategy

	PLAYER B		
PLAYE R A		B1	B2
	A1	2	-1
	A2	-1	0

Rule of Dominance, mixed strategy (without saddle point)

Solve the game whose payoff matrix is given below:

	B				
		I	II	III	IV
A	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

Rule of Dominance, mixed strategy (without saddle point)

Solve the game whose payoff matrix is given below:

	B					
		I	II	III	IV	V
A	I	1	3	2	7	4
	II	3	4	1	5	6
	III	6	5	7	6	5
	IV	2	0	6	3	1

Rule of Dominance, mixed strategy (without saddle point)

In a small town, there are only two stores, ABC and XYZ that handle sundry goods. The total number of customers are equally divided between the two, because the price and quality of goods sold are equal. Both stores have good reputation in the community, and they render equally good customer service. Assume gain to ABC is loss to XYZ and vice-varsa. Both stores plan to run annual pre-diwali sales during the first week of November. Sales are advertised through a local newspaper, radio and TV media. Determine the optimal strategies and the worth of such strategies for both ABC and XYZ. Figures in matrix represent gain or loss to the customers.

	Strategy of XYZ		
Strategy of ABC	Newspaper	Radio	Television
Newspaper	30	40	-80
Radio	0	15	-20
Television	90	20	50

Two breakfast food manufacturers, ABC and XYZ are competing for an increased market share. The payoff matrix, shown in the following table, describes the increase in market share for ABC and decrease in market share of XYZ. Determine optimal strategies for both the manufacturers and the value of the game.

		XYZ			
		Give coupons	**Decrease prices**	**Maintain present strategy**	**Increase advertising**
ABC	**Give coupons**	**2**	**-2**	**4**	**1**
	Decrease prices	**6**	**1**	**12**	**3**
	Maintain present strategy	**-3**	**2**	**0**	**6**
	Increase advertising	**2**	**-3**	**7**	**1**

Solve the following games by using the principle of dominance:

	B1	B2	B3
A1	50	40	28
A2	70	50	45
A3	75	47.5	50
A4	75	50	50

	B1	B2	B3	B4	B5
A1	1	3	2	7	4
A2	3	4	1	5	6
A3	6	5	7	6	5
A4	2	0	6	3	1

Graphical method for 2 *n or m * 2 Games

Solve the game by graphical method.

	B				
A		y1	y2	y3	y4
	x1	19	6	7	5
	x2	7	3	14	6
	x3	12	8	18	4
	x4	8	7	13	-1

Graphical method for 2 *n or m * 2 Games

Solve the game by graphical method.

		B				
A		y1	y2	y3	y4	y5
	x1	-5	5	0	-1	8
	x2	8	-4	-1	6	-5

- **Implications**
- Game Theory provides many insights into the behaviour of oligopolists. For example, it indicates that generating rules for behaviour may take some of the risks out of competition, such as:
- Employing a simple cost-plus pricing method which is shared by all participants. This would work well in situations where oligopolists share similar or identical costs, such as with petrol retailing.
- Implicitly agreeing a 'price leader' with other firms as followers. In the Airline example, firm A may lead and raise price, with B passively following suit. In this case, both would generate revenues of £120.
- Supermarkets implicitly agreeing some lines where price cutting will take place, such as bread or baked beans, but keeping price constant for most lines.
- Generally keeping prices stable (sticky) to avoid price retaliation.

Answer:

I. The best strategy for A is A1cand for B is B3.

II. The maximum of the row minima and minimum of the column maxima are same. This value is referred to as the saddle point.

III. The payoff amount in the saddle point position is called the value of the game.

For this game V= -2, for player A.

Note: the value of the game is always expressed from the point of view of the player whose strategies are listed in the row.

IV. The game is strictly determinable

V. Since the value of the game is not zero, the game is not fair.

Assessment Questions

- Which of the following is an example of a game theory strategy?
  - a. You scratch my back and I'll scratch yours.
    b. If the shoe fits, wear it.
    c. Monkey see, monkey do.
    d. None of the above.

- In game theory, the outcome or consequence of a strategy is referred to as the

  a. Payoff.
  b. Penalty.
  c. Reward.
  d. End-game Strategy.

- The two person zero-sum game means that the Sum of losses to one player equals the sum of gains to other

  a. Sum of losses to one player is not equal to the sum of gains to other
  b. Sum of both the players is equal to zero
  c. Both a, b and c
  d. Both and b
  e. Both a and c

- Game theory models are classified by the
  a. Number of players
  b. Sum of all payoffs
  c. Number of strategies
  d. All of the above

- A game is said to be fair if
  a. Both upper and lower values of the game are same and zero
  b. Upper and lower values of the game are not equal
  c. Upper value is more than the lower value of the game
  d. None of the above

- What happens when maxmin and minmax values of the game are same?
  a. No solution exists
  b. Solution is mixed
  c. Saddle point exists
  d. None of the above

- A prisoners' dilemma is a game with all of the following characteristics except one. Which one is present in a prisoners' dilemma?
    a. Players cooperate in arriving at their strategies.
    b. Both players have a dominant strategy.
    c. Both players would be better off if neither chose their dominant strategy.
    d. The payoff from a strategy depends on the choice made by the other player.

- Which one of the following is a part of every game theory model?
  a. Players
  b. Payoffs
  c. Probabilities
  d. Strategies

- Which of the following is a nonzero-sum game?
  - a. Prisoners’ dilemma
    b. Chess
    c. Competition among duopolists when market share is the payoff
    d. All of the above.

- Which of the following is a zero-sum game?
  - a. Prisoners' dilemma
    b. Chess
    c. students deciding whether or not to cheat
    d. All of the above.

- Which one of the following conditions is required for the success of a tit-for-tat strategy?

  a. Demand and cost conditions must change frequently and unpredictably.
  b. The number of oligopolists in the industry must be relatively small.
  c. The game can be repeated only a small number of times.
  d. Firms must be unable to detect the behavior of their competitors.

Reference Books

- Operations Research Theory and Applications by J.K. Sharma, Macmillan India Ltd.