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UNIT I
MEASURES OF CENTRAL TENDENCY AND DISPERSION

Programme Educational Objectives

Our program will create graduates who:

1. *Will be recognized as a creative and an enterprising team leader.*
2. *Will be a flexible, adaptable and an ethical individual.*
3. *Will have a holistic approach to problem solving in the dynamic business environment.*

Research Methodology & Quantitative Techniques Course Outcomes

CO1-Given a managerial problem and associated frequency distribution data, the student manager will be able to apply descriptive and inferential statistics to facilitate quick and rationale managerial decision making.

CO2-Given the data for two or more variables, the student manager will be able to estimate the strength of the relationship between two variables using ‘Karl Pearson’ and ‘Spearman’s Rank’ correlation coefficient.

CO3-Given the data for two or more variables, the student manager will be able to predict / forecast using as moving averages, regression and time series analysis.

CO4-Given a managerial problem, the student manager will be able to formulate it as ‘research problem’ and also will be able to suggest suitable research methodology to identify workable solutions.

CO5-Given a business Problem/situation, the student manager will be able to develop methods and instruments (questionnaire/ interview schedule) for collection and measurement of qualitative as well as quantitative data using primary and secondary sources from a given sampling framework.

CO6-Given the sample statistics, the student manager will be able to apply Z, t and Chi-square tests to accept or reject the stated hypotheses for making sound decisions.

Learning Objective

- To learn the different measures of central tendency including mean, median and mode
- To learn various methods of calculating the measures of central tendency

ARITHMETIC MEAN – DIRECT METHOD

$$\overline{X} = \frac{1}{N} \sum X \text{ or } \overline{X} = \frac{\sum X}{N}$$

Where,

$\overline{X}$ =Arithmetic Average,

X =Values of the variable,

$\sum$ =Summation or Total,

N =number of items.

EXAMPLE 1

Calculate the Simple Arithmetic Average of the following items by Direct Method:

Size of the item (X)		
20	50	72
28	53	74
34	54	75
39	59	78
42	64	79

ARITHMETIC MEAN – SHORTCUT METHOD

$$\overline{X} = A + \frac{\sum dX}{N}$$

$\overline{X}$ =Arithmetic Average,

A=Assumed Arithmetic Average,

X =Values of the variable,

dX =(X - $\overline{X}$)

N =number of items.

EXAMPLE 2

Calculate the Simple Arithmetic Average of the following items using assumed mean as 50:

Size of the item (X)		
20	50	72
28	53	74
34	54	75
39	59	78
42	64	79

MEAN OF DISCRETE SERIES – DIRECT METHOD

If f1, f2, f3 etc. stand respectively for the frequencies of the values X1, X2, X3 etc.,

$$\bar{X} = \frac{1}{N}\left(f_1X_1 + f_2X_2 + f_3X_3 + \cdots f_nX_n\right)$$

OR

$$\overline{X} = \frac{\sum fX}{N} = \frac{\sum fX}{\sum f}$$

EXAMPLE 3

The following table gives the number of children born per family in 735 families. Calculate the average number of children born per family.

Number of Children Born per Family	Number of Families	Number of Children Born per Family	Number of Families
0	96	7	20
1	108	8	11
2	154	9	6
3	126	10	5
4	95	11	5
5	62	12	1
6	45	13	1

MEAN OF DISCRETE SERIES – SHORTCUT METHOD

If f1, f2, f3 etc. stand respectively for the frequencies of the values X1, X2, X3 etc.,

$$\overline{X} = A + \frac{\sum f dX}{N}$$

Where, $\sum fdX$ = the total of the products of the deviations from the assumed average and the respective frequencies of the items.

EXAMPLE 4

Following data relate to sizes of shoes sold by a store during a given week. Find the average size by the short-cut method assuming mean size as 8.

Size of Shoes	No. of Pairs	Size of Shoes	No. of Pairs
4.5	1	8	95
5	2	8.5	82
5.5	4	9	75
6	5	9.5	44
6.5	15	10	25
7	30	10.5	15
7.5	60	11	4

SIZE (X)	No. (f)	dx = X - 8	fdx
4.5	1	-3.5	-3.5
5	2	-3	-6
5.5	4	-2.5	-10
6	5	-2	-10
6.5	15	-1.5	-22.5
7	30	-1	-30
7.5	60	-0.5	-30
8	95	0	0
8.5	82	0.5	41
9	75	1	75
9.5	44	1.5	66
10	25	2	50
10.5	15	2.5	37.5
11	4	3	12

Σ fdx = 169.5

$$\overline{X} = A + \frac{\Sigma f dX}{N}$$

= 8 + (169.5/457)

= 8 + (0.370)

= 8.370

EXAMPLE 5

The Following table gives the heights of 350 men. Calculate the mean height of the group.

Height in cm	No. of Persons
159	1
161	2
163	9
165	48
167	131
169	102
171	40
173	17

167.89

MEAN OF CONTINUOUS SERIES

$$\overline{X} = \frac{\sum fm}{N} = \frac{\sum fm}{\sum f}$$

Where, m = Midpoint Value of the class interval.

EXAMPLE 6

The following table gives the marks obtained by a set of students in a certain examination. Calculate the average marks per student.

Marks	Number of Students	Marks	Number of Students
10-20	1	60-70	12
20-30	2	70-80	16
30-40	3	80-90	10
40-50	5	90-100	4
50-60	7		

EXAMPLE 7

Calculate the arithmetic average of the following by the direct method

Weekly Wages (in Rupees)	Number of Laborers
11-13	3
13-15	4
15-17	5
17-19	6
19-21	5
21-23	4
23-25	3

EXAMPLE 8 – Open Class Intervals

Calculate the arithmetic mean of the following series.

Marks	No. of Students
< 10	4
10-20	6
20-30	10
30-40	20
40 <	10

EXAMPLE 9 – Open Class Intervals

Calculate the arithmetic mean of the following series.

Weekly wages	No. of Workers
Below 20	10
20-50	20
50-90	40
90-140	15
Above 140	15

EXAMPLE 10 – Step Deviation Method

for simplification of calculations deviations can be further divided by a common factor and if this factor is represented by i

$$\overline{X} = A + \left(\frac{\sum f dX}{N}\right) i$$

The Following table gives the heights of 350 men. Calculate the mean height of the group.

No of persons	1	2	9	48	131	102	40	17
Height in cms	159	161	163	165	167	169	171	173

X	f	dx =X - 167	Step Dev. = dx/2	Tot Dev. = f dx
159	1	- 8	- 4	-4
161	2	- 6	- 3	-6
163	9	- 4	- 2	- 18
165	48	- 2	- 1	-48
167	131	0	0	0
169	102	2	1	102
171	40	4	2	80
173	17	6	3	51
				$\sum$ f dx = 157

$$\overline{X} = A + \left(\frac{\Sigma f dX}{N}\right) i \qquad \overline{X} = 167 + \left(\frac{157}{350}\right) 2 \qquad = 167.89$$

EXAMPLE 11 – Missing Frequency

Find the missing frequency if the arithmetic mean of the series is 16.82.

Marks	Frequency
0-5	10
5-10	12
10-15	16
15-20	U
20-25	14
25-30	10
30-35	8

Solution:

X	f	m	fm
0-5	10	2.5	25
5-10	12	7.5	90
10-15	16	12.5	200
15-20	U	17.5	17.5 U
20-25	14	22.5	315
25-30	10	27.5	275
30-35	8	32.5	260
N = (70 + U)		Σ fm = (1165 + 17.5 U)	

Solution:

$$\overline{X} = \frac{\sum fm}{N} = \frac{\sum fm}{\sum f}$$

16.82 = (1165 + 17.5 U) / (70 + U)

1177.4 + 16.82 U = 1165 + 17.5 U

1177.4 - 1165 = 17.5 U - 16.82 U

12.4 = 0.68 U

U = 18.23

Approx 18

Example 12 - Weighted Mean

A candidate scores the following percentages in an exam – English 46%, Mathematics 67%, Sanskrit 72%, Economics 58%, Political science 53%.

It is agreed to give double weights to marks obtained in English and Mathematics as compared to other subjects.

What is the simple and weighted mean marks scored by the candidate?

MEDIAN

Median is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or in descending order of their magnitudes.

It divides the arranged series in two equal parts.

Median is a position average, whereas the arithmetic mean is a calculated average.

Find out the median of the following items:

5, 7, 9, 12, 10, 8, 7, 15, 21

ITEMS GIVEN	REARRANGED ITEMS
5	5
7	7
9	7
12	8
10	9
8	10
7	12
15	15
21	21

$$M = Size\ of \left(\frac{N+1}{2}\right) th\ item$$

$$M = Size\ of \left(\frac{9+1}{2}\right) th\ item$$

M = Size of 5th item

Find out the median of the following items:

391, 384, 591, 407, 672, 522, 777, 753, 2488 & 1490

ITEMS GIVEN	REARRANGED ITEMS
391	384
384	391
591	407
407	522
672	591
522	672
777	753
753	777
2488	1490
1490	2488

$$M = Size\ of \left(\frac{N+1}{2}\right) th\ item$$

$$M = Size\ of \left(\frac{10+1}{2}\right) th\ item$$

M = Size of 5.5th item

(591+672)/2

M = 1263/2 = 631.5

MEDIAN – DISCRETE SERIES

Given below is the data of wages paid to different people. Find out the median wages paid.

Wages	1000	1500	800	2000	2500	1800
No. of persons	24	26	16	20	6	30

Wages	1000	1500	800	2000	2500	1800
No. of persons	24	26	16	20	6	30

Wages	800	1000	1500	1800	2000	2500
No. of persons (f)	16	24	26	30	20	6
Cumulative Frequency	16	40	66	96	116	122

$$M = Size\ of \left(\frac{N+1}{2}\right) th\ item$$

$$M = Size\ of \left(\frac{122+1}{2}\right) th\ item$$

M = Size of 61.5th item
= 1500

MEDIAN – DISCRETE SERIES

Given below is the data of grades scored by students of a class in an exam. Grades range from A+ as BEST and C as WORST. Find out the median grade of the class.

Grades	A+	C+	B	B+	A	C
No. of students	5	9	20	14	6	6

MEDIAN – CONTINUOUS SERIES

$$Median = L + \frac{\frac{N}{2} - c.f.}{f} * i$$

L = the lower limit of the median class

N/2 = middle number

c.f. = the cumulative frequency of the class preceding the median class

f = the frequency of the median class and

i = the magnitude of the median class interval

MEDIAN – CONTINUOUS SERIES

$$M = l_1 + \frac{l_2 - l_1}{f_1}(m - c)$$

M = the value of the median

l_1 & l_2 = lower and upper limit of the class in which median lies

f_1 = frequency of the median class

m = middle number whose value is median (N/2)

c = cumulative frequency of the class preceding the median class

MEDIAN – CONTINUOUS SERIES

Find the median of the following distribution

Class intervals (Rs.)	Frequency	Class intervals (Rs.)	Frequency
1-3	6	9-11	21
3-5	53	11-13	16
5-7	85	13-15	4
7-9	56	15-17	4

CLASS	FREQ.	CUM. FREQ.
1 – 3	6	6
3 – 5	53	59
5 – 7	85	144
7 – 9	56	200
9 – 11	21	221
11 – 13	16	237
13 - 15	4	241
15 - 17	4	245

$$Median = L + \frac{\frac{N}{2} - c.f.}{f} * i$$

$$Median = 5 + \frac{122.5 - 59}{85} * 2$$

$$Median = 6.494 = 6.5$$

$$M = l_1 + \frac{l_2 - l_1}{f_1}(m - c)$$

$$M = 5 + \frac{7 - 5}{85}(122.5 - 59) = 6.5$$

$$\frac{N}{2} = \frac{245}{2} = 122.5$$

MEDIAN – CONTINUOUS SERIES

Find the median age of the following distribution

Age	No. of Persons	Age	No. of Persons
55-60	7	35-40	30
50-55	13	30-35	33
45-50	15	25-30	28
40-45	20	20-25	14

ANSWER : MEDIAN = 35.83

WHEN LESS THAN VALUES ARE GIVEN

Find the median of the following data

Value	Frequency	Value	Frequency
Less than 10	4	Less than 50	96
Less than 20	16	Less than 60	112
Less than 30	40	Less than 70	120
Less than 40	76	Less than 80	125

WHEN MORE THAN VALUES ARE GIVEN

Find the median of the following data

Size	Frequency
More than 50	0
More than 40	40
More than 30	98
More than 20	123
More than 10	165

.7

WHEN ONLY MID VALUES ARE GIVEN

Find the median of the following data

Mid Value	Frequency	Mid Value	Frequency
115	6	165	60
125	25	175	38
135	48	185	22
145	72	195	3
155	116		

ANSWER : MEDIAN = 153.8

MODE

- *Mode is the value in a series which occurs most frequently.*
- *In a frequency distribution mode is that variate which has the maximum frequency.*

Examples

Average size of the shoe sold in a shop is 7.

Average height of an Indian male is 5 feet 6 inches.

Average size of the shirt sold in a ready made garment shop is 40.

MODE – INDIVIDUAL OBSERVATIONS.

Weight of 10 persons were taken randomly. Results were recorded in the below table. Calculate the Modal weight.

Sr. No.	Weight in Pounds	Sr. No.	Weight in Pounds
1	120	6	130
2	130	7	132
3	135	8	132
4	130	9	135
5	140	10	141

MODE – GROUPING METHOD.

Size	Frequency	Size	Frequency
5	48	13	52
6	52	14	41
7	56	15	57
8	60	16	63
9	63	17	52
10	57	18	48
11	55	19	40
12	50		

Size	Frequency	Column of two	Column of two leaving the first	column of three	column of three leaving the first	column of three leaving the first two
X	f					
	(I)	(II)	(III)	(IV)	(V)	(VI)
5	48	100		156		
6	52		108		168	179
7	56	116				
8	60		123	180		
9	63	120			175	
10	57		112			162
11	55	105		157		
12	50		102		143	
13	52	93				150
14	41		98	161		
15	57	120			172	
16	63		115			163
17	52	100				
18	48			140		
19	40		88			

Size	Frequency	Column of two	Column of two leaving the first	column of three	column of three leaving the first	column of three leaving the first two
X	f					
	(I)	(II)	(III)	(IV)	(V)	(VI)
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6	52		108		168	
7	56	116				179
8	60		123	180		
9	63	120			175	
10	57		112			162
11	55	105		157		
12	50		102		143	
13	52	93				150
14	41		98	161		
15	57	120			172	
16	63		115			163
17	52	100		140		
18	48		88			
19	40					

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13	52	93				150
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18	48		88			
19	40					

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	(I)	(II)	(III)	(IV)	(V)	(VI)
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12	50	105	102		143	
13	52					150
14	41	93	98	161		
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16	63	120	115			163
17	52			140		
18	48	100	88			
19	40					

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	(I)	(II)	(III)	(IV)	(V)	(VI)
5	48					
6	52	100	108	156	168	
7	56					179
8	60	116	123			
9	63			180	175	
10	57	120	112			162
11	55					
12	50	105	102	157	143	
13	52					150
14	41	93	98			
15	57			161	172	
16	63	120	115			163
17	52					
18	48	100	88	140		
19	40					

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	(I)	(II)	(III)	(IV)	(V)	(VI)
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6	52		108		168	179
7	56	116		180		
8	60		123		175	162
9	63	120		157		
10	57		112		143	150
11	55	105		161		
12	50		102		172	163
13	52	93		140		
14	41		98			
15	57	120				
16	63		115			
17	52	100				
18	48		88			
19	40					

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X	f					
	(I)	(II)	(III)	(IV)	(V)	(VI)
5	48	100		156		
6	52		108		168	
7	56	116				179
8	60		123	180		
9	63	120			175	
10	57		112			162
11	55	105		157		
12	50		102		143	
13	52	93				150
14	41		98	161		
15	57	120			172	
16	63		115			163
17	52	100		140		
18	48		88			
19	40					

MODE – GROUPING METHOD.

Size	Frequency
4	2
5	5
6	8
7	9
8	12
9	14
10	14
11	15
12	11
13	13

MODE – CONTINUOUS SERIES.

In a continuous series first the modal class is identified by grouping method and then the below formula is used to find out the MODE.

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} * i$$

Where,

l = Lower limit of modal class.

f_m = Frequency of modal class.

f_1 = Frequency of class preceding modal class.

f_2 = Frequency of class succeeding modal class.

i = width of modal class.

MODE – CONTINUOUS SERIES.

The following table gives the length of life of 150 electric lamps. Find the mode.

Life of Lamps (hours)	Frequency
0 to 400	4
400 to 800	12
800 to 1200	40
1200 to 1600	41
1600 to 2000	27
2000 to 2400	13
2400 to 2800	9
2800 to 3200	4

(Answer: Mode = 1226.67 hours)

MEAN, MEDIAN & MODE.

Find the value of Mode from the following data (from Mean and Median)

Size of Item	Frequency
100-110	4
110-120	6
120-130	20
130-140	32
140-150	33
150-160	17
160-170	8
170-180	2

(Answer: Mode = 140.05)

MEAN, MEDIAN & MODE.

Empirical relationship.

$$MODE = 3\ MEDIAN - 2MEAN$$

MEAN, MEDIAN & MODE.

(A) Given, Mean = 20, Mode = 15, find the value of Median

(B) Given Mode = 25, Median = 20, find the value of Mean

(Answer: Median = 18.3)
(Answer: Mean = 17.5)

References and Suggested Readings

Fundamentals of Statistics by S.C. Gupta

Statistics Methods by S.P.Gupta